

## Abstract Title Page

**Title:** *Fostering First-Graders' Fluency with Basic Subtraction Combinations*

Authors: Arthur J. Baroody, David J. Purpura, Michael D. Eiland, and Erin E. Reid  
(University of Illinois at Urbana-Champaign)

## Abstract Body

**Background:** Subtraction combinations are particularly challenging for children to learn (Kraner, 1980; Smith, 1921; see Cowan, 2003, for a review). Even relating subtraction to addition (e.g., Think of  $8-5=?$  as “ $5+\text{what number}=8?$ ”) frequently does not help, because the complement principle (e.g., If  $5+3=8$ , then  $8-5=3$ ) is not obvious to primary-level children (Baroody, 1999; Baroody, Ginsburg, & Waxman, 1983; Canobi, 2004, 2005, 2009; Henry & Brown, 2008; Putnam, deBettencourt, & Leinhardt, 1990).

Three adjustments were made in the present experimental training of the subtraction-as-addition strategy to make its rationale (the complement principle) more apparent and intelligible. (a) A subtraction item such as  $8-5=?$  was relating to  $?+5=8$ , not  $5+?=8$ . As the addend 5 appears in the same location in the subtraction and the addition equation, it should better draw children’s to the fact that addition and subtractions complements have the same parts and whole. (b) Some activities modeled “empirical inversion,” which theoretically should help children understand the complement principle (Baroody, Torbeyns, & Verschaffel, 2009). Such activities involved moving to 3 on a 1 to 20 number list, adding 5, and predicting the new location on a number list (also symbolically represented as  $3+5=?$ ). After determining the new location (also symbolically represented as  $3+5=8$ ), the child was next asked to predict the location on a number list if they then backtracked 5 or essence undid the addition previous addition of 5 (also symbolically represented as  $8-5=?$ ). The solution to the follow-up subtraction item underscored that taking away 5 after adding 5 brought you back to 3 again (also symbolically represented as  $8-5=3$ ). (c) In order to further underscore the connections between related equations and foster part-whole understanding, which theoretically is the basis for understanding various relations between addition and subtraction (Briars & Larkin, 1984; Canobi, 2005; Resnick, 1983, Riley, Greeno, & Heller, 1983; Piaget, 1957; UCSMP, 2005), the *total* for each complementary addition and subtraction equation was labeled the “whole,” each *part* was labeled “part,” and the corresponding components in each equation had a distinct color.

**Research Questions:** Would the group receiving the *experimental subtraction-as-addition* training outperform the *control* group, which received training on a different reasoning strategy involving 8s or 9s, on both practiced and unpracticed subtraction combinations and (2) a group that receiving *unstructured subtraction practice* on at least the unpracticed subtraction items. (Transfer to unpracticed items indicates the successful learning of a general reasoning strategy.)

**Setting:** A total of 75 first graders (6.1 to 7.6 years old, mean=6.6) from five schools in two school districts serving a mid-sized mid-western community participated in the study. See Table 1 for details.

**Intervention:** The preparatory training (Stages I and II) is detailed in Table 2 of Paper 1. The training by conditions (Stages III to V) is delineated in Table 2. The experimental subtraction-as-addition training is illustrated in Figures 1 to 7. The preparatory training (Stages I and II in Table 2) was identical for all participants and that the training differed by condition in Stage III to V.

**Research Design:** All children in the sample pool simultaneously received the 7.5-week long preparatory (Stage I and II) training. During this time, children were pretested on the TEMA-3.

After the completion of the preparatory training, participants were individually administered a preliminary computer-based mental-addition screening test that served to gauge fluency with the easiest sums: adding with 0 and 1 and the doubles (see Authors, xxxx, for details). Participants fluent on more than half the items in these combinations families were eligible for the present study and were administered the computer-based mental-arithmetic pretest to gauge fluency with subtraction and more difficult addition combinations. Participants who were fluent on less than half the subtraction and adding with 8 or 9 items were then randomly assigned by class to structured learning/practice of subtraction-as-addition reasoning strategy, structured learning/practice of use-a-ten reasoning strategy, or a unstructured practice of subtraction and  $n+8/8+n$  and  $n+9/9+n$  combinations. The computer-assisted experimental interventions were conducted simultaneously, and each lasted 12 weeks. Both preparatory training and experimental interventions involved one-to-one, 30-minute sessions twice per week. All project training was conducted at project computer stations in a hallway outside a child's classroom or in a room dedicated to the project. Pull outs occurred in non-literacy time blocks, including mathematics instruction and play time. All participants were re-tested on the mental arithmetic items two weeks after the training to gauge retention. Project personnel implemented all testing and training procedures. Positive assent was obtained for each testing and training sessions.

**Data Collection and Analysis:** The test of mental arithmetic fluency included five categories of items: (a) practiced subtraction items (7–5, 9–5, 10–7, 11–6, 11–7, 12–9, 14–7); (b) unpracticed (transfer) subtraction items (9–6, 10–6, 11–5, 11–8, 12–7, 13–7, 16–8), (c) practiced addition complements of (practiced and unpracticed) subtraction items (3+9, 4+5, 4+6, 5+6, 5+7, 6+5, 6+7, 7+7, 8+8), (d) practiced *use-a-ten* items (5+8, 7+8, 7+9, 8+4, 8+7, 9+5, 9+6, 9+9); (e) unpracticed (transfer) *use-a-ten* items (4+8, 5+9, 6+8, 8+5, 8+6, 8+9, 9+7, 9+8). Note that category *a* items were practiced by the subtraction-as-addition and the unstructured-practice groups; category *c* items, by the subtraction-as-addition only; and the category *d* items, by the use-a-ten and the unstructured-practice groups. None of the groups practiced category *b* and category *e* items, and so category *b* served as transfer items for subtraction-as-addition and the unstructured-practice groups, and category *e* served as transfer items for the use-a-ten and the unstructured-practice groups. The testing was done in the context of a computer game.

As the two primary groups (subtraction and use-a-ten) targeted different types of skills each was used as a control group for the other. The unstructured-practice group was also utilized as an active instructional comparison group in both sets of analyses to determine if the structured discovery practice resulted in better outcomes than just simply practicing the items. Analyses of fluency were done using the proportion correct by a child on a test. ANCOVAs, using pretest mental-arithmetic fluency, pretest TEMA-3 pretest, and age as the covariates, were used to compare posttest performance of each group on targeted practiced and unpracticed combinations.

## Findings / Results:

***The impact of structured subtraction-as-addition and unstructured practice.*** As predicted, the structured subtraction-as-addition group significantly outperformed the use-a-ten (control) group,  $F(1, 45) = 31.68, p < .001$ , *Hedge's g* = 1.46, but not the unstructured-practice group,  $F(1, 45) = .68, p = .414$ , *Hedge's g* = .22. However, the effect size between these two groups favoring the structured group was—especially given that this was a delayed posttest result—educationally meaningfully. Additionally, the unstructured-practice group significantly outperformed the control group,  $F(1, 45) = 13.21, p = .001$ , *Hedge's g* = .95.

As predicted for the *unpracticed* subtraction items, the structured subtraction group significantly outperformed the control group,  $F(1, 45) = 4.07, p = .050$ , *Hedge's g* = .50 and the unstructured-practice group,  $F(1, 45) = 4.56, p = .038$ , *Hedge's g* = .55. The unstructured-practice group did not outperform the control group,  $F(1, 45) = .25, p = .619$ , *Hedge's g* = -.12.

For the *practiced* addition complements of the subtraction items, the structured subtraction-as-addition group significantly outperformed the control group,  $F(1, 45) = 18.40, p < .001$ , *Hedge's g* = 1.04, and the unstructured-practice group,  $F(1, 45) = 14.25, p < .001$ , *Hedge's g* = .77. The unstructured-practice group did not significantly outperform the control group,  $F(1, 45) = 1.31, p = .259$ , *Hedge's g* = .26.

**The impact of structured use-a-ten and unstructured practice.** For *practiced use-a-ten*, the structured use-a-ten group significantly outperformed the subtraction-as-addition (control) group,  $F(1, 45) = 9.03, p = .004$ , *Hedge's g* = .78, but not the unstructured-practice group,  $F(1, 45) = 1.18, p = .284$ , *Hedge's g* = -.26. Additionally, the unstructured-practice group significantly outperformed the control group,  $F(1, 70) = 18.79, p < .001$ , *Hedge's g* = .99.

The structured use-a-ten group did not significantly outperform the control group for *unpracticed use-a-ten-items*,  $F(1, 45) = .36, p = .550$ , *Hedge's g* = .15, or the unstructured practice group,  $F(1, 45) = .00, p = .996$ , *Hedge's g* = -.02. The unstructured practice group did not significantly outperform the control group,  $F(1, 45) = 1.52, p = .224$ , *Hedge's g* = .28.

**Conclusions:** Regarding the delayed posttest results with the *practiced subtraction* combinations, the structured subtraction-as-addition group, which received supplemental subtraction instruction/practice, significantly outperformed the use-a-ten group, which received regular classroom subtraction instruction/practice. Moreover, as indicated by a small, but appreciable, effect size (Cohen, 1992), the structured subtraction group outgained the unstructured-practice group, which received supplemental subtraction practice but not training on how subtraction is related to addition. The unstructured practice group also significantly outperformed the use-ten group. For the *subtraction combinations not practiced* by any group, the structured subtraction out gained the other groups at a marginally significant level and with a medium effect size indicative of instructional effectiveness (IES, 2011). The other groups did not differ on the unpracticed subtraction items.

The pattern of these results provides evidence of the efficacy of the structured subtraction intervention. Although either structured *or* unstructured supplemental practice is more effective than typical classroom training in promoting fluency with practiced subtraction items, the structured training provided additional benefit over haphazard supplemental practice with such items. More importantly, the structured subtraction—but not the unstructured practice—apparently enabled first graders to learn the subtraction-as-addition reasoning strategy that they could fluently apply to unpracticed subtraction combinations. This transfer was achieved *despite* only 34 repetitions for each practiced subtraction item and 26 repetitions for each related *addition complement*—substantially less practice than thousands of repetitions per item necessary to achieve memorization (by rote) of a basic fact specified by earlier models/computer simulations of arithmetic learning (e.g., Shrager & Siegler, 1998; Siegler & Araya, 2005; Siegler & Jenkins, 1989).

The structured subtraction-as-addition group, which practiced the addition complements of subtraction, not unexpectedly were more fluent on such items at the delayed posttest than the other groups, which did not practice the items. It is unclear whether this superior performance was due to the structured training (e.g., relating addition and subtraction to each other and part-

whole relations), practicing related sums and differences together (e.g., the facilitating effect of practicing another related combination—one with same parts and whole), or merely extra addition practice. The latter, however, seems unlikely for two reasons. One is that addition items were practiced only 26 times each during the intervention. Another is that the unstructured-practice group (which practiced subtraction and unrelated addition item) surprisingly became appreciably more fluent with the addition complements (which they did not practice) than the use-ten (control) group (which practiced only combinations involving the addition of 8, 9, or 10). A possible contributing factor is that practicing  $14-7=7$ , for instance, facilitated learning the unpracticed addition combination  $7+7$ . Although clearly, in need of further systematic research, it may be that learning subtraction combinations can impact learning of addition complements as well as vice versa.

The results regarding the structured use-a-ten training were disappointing. Although this training resulted in significant improvements on practiced  $n+8/8+n$  and  $n+9/9+n$  items, so did unstructured practice. Moreover, it was not more effective in promoting fluency with unpracticed  $n+8/8+n$  and  $n+9/9+n$  items than the other conditions. Findings suggest that different strategies and instructional techniques may be necessary for the different relational families and these findings provide a starting point with which to build knowledge in this area.

The results of the structured subtraction-as-addition training, if not the structured use-a-ten training, are consistent with the recommendations of the NRC (2001) that Phase 2—learning reasoning strategies—can be accelerated by directly teaching reasoning strategies, if done conceptually. Likewise, the present results are consistent the NMAP (2008) conclusion and the number sense view that structured practice can be an effective instructional tool in promoting the learning of the relations and promoting fluency (Phase 3). Although both structured and unstructured practice were more effective in improving fluency with practiced subtraction items than typical first-grade mathematics instruction, only the structured training was effective in promoting the learning of a reasoning strategies that could be applied efficiently and effectively to non-practiced subtraction items.

Several features of the structured subtraction training may have contributed its success:

1. *Connecting the complementary relation between addition and subtraction to Highlighting empirical inversion—children’s informal tendency to view these operations as separate or unrelated processes*—can help them to see that addition and subtraction are, in fact, inter-related operation—that is, complementary. For example, figuring out that  $3+5$  is 8 and then undoing this incrementing process (subtracting the same amount 5) to arrive at 3 again may have been particularly helpful in seeing that  $3+5=8$  and  $8-5=3$  are inter-related.
2. *Labeling and color-coding the common whole and parts in each juxtaposed equation* can further help children see that addition and subtraction are complementary.
3. *Keeping the second addend or part in the same position* may facilitated the effect of feature 1 by underscoring the same amount was added and then taken away from another and feature 2 by highlighting the two equations have a common part.

Fostering an understanding of the complementary relation between addition and subtraction provides a basis for meaningfully learning and applying the subtraction-as-addition reasoning strategy. Although further research is clearly needed to evaluate the impact of these features separately and in combination, the present results suggest that the meaningful learning of a subtraction-addition strategy, which children can be apply to unpracticed subtraction combinations can significantly reduce the amount of time and practice needed to achieve fluency with basic subtraction combinations.

## Appendices

### Appendix A. References

- Allardice, B. S., & Ginsburg, H. P. (1983). Children's learning problems in mathematics. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 319–349). New York: Academic Press.
- Dev, O. C., Doyle, B. A., & Valente, B. (2002). Labels needn't stick: "At-risk" first graders rescued with appropriate intervention. *Journal of Education for Students Placed At Risk*, 7, 327–332.
- Fuchs, L.S., Fuchs, D., Hamlet, C. L., Powell, S. R., Capizzi, A. M., & Seethaler, P. M. (2006). The effects of computer-assisted instruction on number combination skill in at-risk first graders. *Journal of Learning Disabilities*, 39, 467–475.
- Fuson, K. C., & Brinko, K. T. (1985). The comparative effectiveness of microcomputers and flash cards in the drill and practice of basic mathematics facts. *Journal for Research in Mathematics Education*, 16, 225–32.
- Geary, D.C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology*, 49, 363–383.
- Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin*, 114, 345–362.
- Geary, D. C. (1996). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association. Originally published 1994.
- Geary, D. C. (2003). Arithmetic development: Commentary on chapters 9 through 15 and future directions. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 453–464). Mahwah, NJ: Erlbaum.
- Hativa, N. (1988). Sigal's ineffective computer-based practice of arithmetic: A case study. *Journal for Research in Mathematics Education*, 19, 195–214.
- Henry, V., & Brown, R. (2008). First-grade basic facts. *Journal for Research in Mathematics Education*, 39, 153–183.
- Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). Arithmetic fact mastery in young children: A longitudinal investigation. *Journal of Experimental Child Psychology*, 85, 103–119.
- Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, D.C.: National Academy Press.

- Kulik, C-L. C., & Kulik, J. A. (1991). Effectiveness of computer-based instruction: An updated analysis. *Computers in Human Behavior*, 7, 75–94.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics: Standards 2000*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*. Reston, VA: Author.
- National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, D.C.: U.S. Department of Education.
- Russell, R. L, & Ginsburg, H. P. (1984). Cognitive analysis of children's mathematics difficulties. *Cognition and Instruction*, 1, 217–244.

## Appendix B. Tables and Figures

Table 1

*Participant characteristics by condition*

		Training Condition		
		Structured Subtraction	Structured Use-a-Ten	Unstructured Practice
Age range		6.1 to 7.1	6.2 to 7.2	6.2 to 7.6
Mean (SD)		6.6 (0.3)	6.6 (0.3)	6.7 (0.3)
Median age		6.6	6.6	6.8
Number of boys / girls		14 / 11	17 / 8	12 / 13
TEMA-3 range		92 to 137	89 to 142	75 to 125
Mean (SD)		107.2 (11.7)	106.8 (13.2)	103.5 (12.1)
Median TEMA-3		103	105	105
Free/Reduced lunch eligible		8	10	5
Black/Hispanic/Multiracial		8	7	7
Family History	Single-parent	3	3	1
	Parent under 18	0	1	0
	Parents w/o HS	0	0	0
	ESL	4	2	3
Medical/ Developmental Condition	Birth complications	0	0	0
	Visual impairment	0	1	1
	Language delay	0	0	0
	Speech services	1	0	0
	Spina bifida	0	0	0
Behavioral Condition	ADHD	0	0	0
	Aggressive	1	2	0
	Passive/withdrawn	1	1	3



Table 2

*Experimental Mental-Arithmetic (Stages III to V) by Condition*

Stage/ Set	Computer Game <sup>a</sup>	Structured subtraction as addition	Control (Structured use-a-ten)	Unstructured subtraction (and add-with-8 or -9) practice
III / A	<i>Castle Wall</i> (feedback on horizontal # line)	Solve addition item such as $3+9$ ; usually followed by solving a related subtraction item $12-9$ .	Solve $10+n/n+10$ item such as $7+10$ ; usually followed by a related $8+n/n+8$ or $9+n/n+9$ such as $7+8$ or $7+9$ .	Subtraction items such as $12-9$ and $8+n/n+8$ or $9+n/n+9$ solved in haphazard order.
	<i>Train Game</i> (feedback on vertical # line)			
III / B	<i>Does It Help?</i> (Possible helper and target items presented successively)	Solve an addition item such as $2+5$ (or $5+7$ ); then asked if it helps solve a subtraction item such as $7-5$ (yes for $2+5$ and no for $5+7$ ).	Solve a $10+n/n+10$ item such as $6+10$ ; then asked if it helps solve $8+n/n+8$ or $9+n/n+9$ such as $6+9$ (yes) or $7+9$ (no).	Solve one item and asked if a second had the same answer—e.g., $11-6$ & $9-5$ (no), $8+7$ & $7+8$ or $12-9$ & $10-7$ (yes).
	<i>Wall Help?</i> ? (All possible helper items presented first block; all target items, presented in a second block)	Addition items such as $4+5$ and $5+7$ solved first and sums arranged sequentially as part of a wall. Subtraction items such as $9-5$ presented; child asked which sum in the wall helps.	A block of $10+n/n+10$ items is solved first and sums arranged sequentially as part of a wall. $8+n/n+8$ or $9+n/n+9$ items such as presented; child asked which sum in the wall helps.	A block of items is solved in haphazard order first and sums arranged sequentially as part of a wall. Child asked if item in the wall helps (has the same answer as) an item from second block.
IV / A	<i>Timed Monkey?</i> (Possible helper and target items presented successively)	Monkey starts at branch 0 and, for $7+6$ , e.g., swings to branch 7, asked to what branch monkey will be if swings 6 more. Related subtraction follows: If at 13, where will monkey be if swings back 6.	Mocha Monkey swings 7 branches and then 10 more, where will she land? Cocoa Monkey swings 7 branches and then 9 (or 8) more, where will she land?	Mocha Monkey swings 7 branches and then 9 more, where will she land? Cocoa Monkey swings 9 branches and then 9 more, where will she land?
	<i>Clocked Choice</i> (timed version of <i>Does It Help?</i> )	After determining sum (e.g., $4+7=11$ ), asked if, helps answer $11-7$ , $10-7$ , $11-6$ , or None. Feedback indicated that both $4+7$ and $11-7$ have the same whole 11 and same part 7 or that an incorrect choice did not. Then asked to answer the subtraction item. Feedback high-lighted parallel part-whole aspects.	Solved a $10+n/n+10$ item such as $10+7$ and provided feedback. Then asked what which $8+n/n+8$ or $9+n/n+9$ was 1 (or 2) smaller than the $10+n/n+10$ item (e.g., The answer to which problem below 1 smaller than $10+7=17$ : $7+8$ , $7+9$ , $9+9$ , or None?)	Solve one item and asked which of three choices (or none) had the same answer.

IV / B	<i>Lost Puppies</i>	Asked if a puppy with a dog tag consisting of an addition item was such as $2+5$ belonged to part-part family (one part 2; other part 5). Determined sum; feedback indicated that 7 was correct because the whole 7 has parts 2 and 5. Asked if a subtraction item such as $7-5$ belongs to the same family with a part of 2 and part of 7. After feedback, child solved for difference.	Asked if a puppy with a dog tag consisting of a $10+n/n+10$ item such as $7+10$ belonged to the 16 family. Feedback for a correct answer of “No” indicated that it was not the lost puppy and congratulated the child for not taking someone else’s puppy. Then asked to indicate the sum of $7+10$ . The same procedure was followed for $7+9$ .	Asked if a puppy with a dog tag consisting of an addition or a subtraction item had a particular answer
	<i>Timed Train</i>	Same as Train Game, except with a clock and increasingly restrictive time limits.		
V / A	<i>Dirt Bike</i>	Block of practiced addition items and then block of practiced subtraction items solved.	Block of practiced $10+n/n+10$ items and then block of practiced $8+n/n+8$ or $9+n/n+9$ items solved.	Practiced addition and subtraction items practiced in haphazard order.
	<i>Long Jump</i>	Practiced addition and subtraction items practiced in haphazard order.	Practiced $10+n/n+10$ , $8+n/n+8$ , and $9+n/n+9$ items practiced in haphazard order.	Practiced addition and subtraction items practiced in haphazard order.
V / B	<i>Puppy Choice</i>	Practiced addition and subtraction items practiced in haphazard order.	Practiced $10+n/n+10$ , $8+n/n+8$ , and $9+n/n+9$ items practiced in haphazard order.	Practiced addition and subtraction items practiced in haphazard order.
	<i>Treasure Hunt</i>			
	<i>Car Race or Fire truck</i>			

<sup>a</sup>Each game was played 4 times, except the Stage V / Set B games, which were each played twice.

Table 3

*Combinations Practiced by Condition*

Type of Combination Item	Structured Subtraction Training		Structured Use-a-Ten Training		Unstructured Subtraction (and Add-with-8 or -9 Training	
	Practiced Items	Unpracticed Items	Practiced Items	Unpracticed Items	Practiced Items	Unpracticed Items
Subtraction items	7–5, 9–5, 10–7 11–6, 11–7, 12–9, 13–6, 14–7	9–6, 10–6, 11–5, 11–8, 12–7, 13–7, 16–8	-	-	7–5, 9–5, 10–7 11–6, 11–7, 12–9, 13–6, 14–7, 10–3*, 13–6*, 13–2*	9–6, 10–6, 11–5, 11–8, 12–7, 13–7, 16–8
Addition complements to practiced/unpracticed subtraction items	2+5*, 3+6*, 3+7*, 3+8*, 3+9, 4+5, 4+6, 4+7*, 5+6, 5+7, 6+5, 6+7, 7+6, 7+7, 8+8	-	-	-	-	-
Add-with-8 or -9 items	-	-	5+8, 6+9, 7+8, 7+9, 8+4, 8+7, 9+5, 9+6	4+8, 5+9, 6+8, 8+5, 8+6, 8+9, 9+7, 9+8	5+8, 6+9, 7+8, 7+9, 8+4, 8+7, 9+5, 9+6	4+8, 5+9, 6+8, 8+5, 8+6, 8+9, 9+7, 9+8
Add-with-10 aids for adding with 8 or 9	-	-	2+10 to 8+10 & 10+3 to 10+9*	-	-	-
Filler items	-	-	-	-	2+6, 3+5 10–3, 13–2	-

**Note 1.** The structured subtraction group and the unstructured subtraction (and add-with-8 or -9) group practiced the same subtraction combinations and did so the same number of times.

**Note 2.** The structured use-ten group and the unstructured subtraction group (and add-with-8 or -9) practiced the same add-with-8 or -9 combinations and did so the same number of times.

**Note.** An asterisk indicates practiced but not pre- or post-tested.